



# Technical Paper

## Utilization of Alias Free Polyreference for Mixed Mode Structures

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## ABSTRACT

Alias Free Polyreference (AFPOLY), a recent advancement in numerical modal parameter extraction, provides a tool which can accommodate mixed mode results such as high and low damping without the need to iterate on multiple frequency ranges. This technique provides a more consistent modal dataset since all modal parameters can be extracted in a single solution. It also prevents “aliasing” of modal characteristics from out-of-band modes which tend to contaminate those modes which are in the band of interest.

Time-domain polyreference, a modal extraction algorithm developed over 20 years ago, has been widely accepted as the “gold-standard” in most experimental modal analyses conducted since the late 1980s. Implementation of AFPOLY on aerospace structures has been shown to provide a method which can surpass the ability of classic time-domain polyreference to readily extract mixed mode results. Examples of aerospace structures which exhibit a combination of heavily damped and lightly damped modes are presented in order to contrast these modal extraction approaches.

## INTRODUCTION

Various methods of modal extraction and parameter estimation (modal frequencies, damping and shapes) have been around for over 30 years. Prior to the 1970’s, most modal parameter estimation was performed using some form of sinusoidal resonant frequency or normal mode testing. In some cases manual curve fitting methods were applied for frequency and damping estimates such as half-power points [1] or Kennedy-Pancu plots [2] which used the coincident and quadrature components of response measurements relative to an applied excitation force. Numerical methods which allowed for the extraction of modal parameters from test measured frequency response functions (FRF) were developed in the 1970’s and gained wide acceptance during the 1980’s as the International Modal Analysis Conference (IMAC) came into existence and provided a venue for the sharing and development of new ideas. These methods took advantage of the digital data that became available and the improved processes for developing FRF from test measurements. Some common numerical methods which were made available are listed in Table 1. This is by no means a complete list, and it does not attempt to show all methods which employ similar methodologies.

Table 1.

Parameter Estimation Technique	Order	Domain
Peak Response	SDOF	Frequency
Circle Fit	SDOF	Frequency
SDOF Polynomial	SDOF	Frequency
Complex Exponential	MDOF	Time
Least Squares Complex Exponential	MDOF	Time
Time Domain Polyreference	MDOF	Time
Frequency Domain Polyreference	MDOF	Frequency
Ibrahim Time Domain	MDOF	Time
Eigenvalue Realization Algorithm	MDOF	Time
Rational Fraction Polynomial	MDOF	Frequency

The earliest of these curve fitting techniques were single degree of freedom (SDOF) methods and typically derived estimates from single FRF. Examples of these methods are SDOF polynomial (frequency domain) and complex plane circle fit which took advantage of a numerical implementation of Kennedy-Pancu. These methods were typically more tedious to utilize and did not provide a global estimate of the modal parameters. The early 1980’s saw the development of several multiple degree of freedom (MDOF) parameter estimation techniques as

well as those which took advantage of multiple reference FRF data. Perhaps the most well known and widely used of these was the time domain Polyreference [3, 4] method which came into existence in 1982.

## THE “GOLD STANDARD”

Time Domain Polyreference (TDPOLY) rapidly developed into the most widely used modal parameter estimation algorithm employed in commercial software. It provided robust modal estimates and was easily implemented on small computer systems that could be easily used in a large variety of test locations. There were some early issues that made TDPoly somewhat cumbersome to use and interpret results. Numerical lists of pole estimates had to be sorted through to select those poles which seemed to be the best for a particular data set. Error charts provided modest help in guiding the practitioner to the proper pole selection, but it was still a tedious process. But as usage increased, so did enhancements to results interpretation tools that improved its ease of use. A modal confidence factor [5] was one interpretation enhancement that was added which was quickly followed with the addition of the stability plot [6]. The stability plot allowed identification of those poles which were converging on a non-varying solution. Various commercial implementations of these interpretive aids can be seen in software which is in wide use. These are still used today while visual enhancements have been added to improve user interaction. The ability to plot measured FRF and various mode indicator functions (MIF) along with the polyreference stability plot allowed users to see and interact with the parameter estimation process resulting in better modal parameter extraction. Figure 1 shows an example of a stability plot that is typical for use in the polyreference process. This type of visual implementation allows the modal test engineer to see which numerical estimates do a good job of matching the test data. In this case, the lightly damped structure exhibits quickly stabilizing poles as indicated by the box sizes. The use of the modal confidence factor (MCF) in the interpretation process has the effect in this case of allowing computational poles to be cleaned up from the stability plot as shown in Figure 2. Since the structure is lightly damped, the MCF is an effective tool (acting like a visual filter) for improving the appearance of the stability plot. Even in the presence of high damping (with low noise), TDPoly could be used to provide good modal estimates as seen in Figure 3.

The use of the TDPoly method was wide spread in all industries, with particular acceptance in the aerospace industry where many test articles were found to be fairly linear and lightly damped (airplanes and satellites for example). In some cases, it was found that TDPoly was notably difficult to use when the damping of a test article was substantial [7] in the presence of noise in the measurement data. Part of this was observed to be the use of the interpretive tools (i.e. MCF) along with the stability plot. As a result, when some “difficult” structures were tested, other modal parameter estimation tools were sometimes used, such as those which incorporate a frequency domain polynomial estimate. On structures which exhibited a large number of modes, or high modal density, it was also sometimes necessary to analyze the data in frequency bands with an overlapping section.

What is usually observed in the use of TDPoly is that a number of computational modes are exhibited in the parameter estimation process. These computational modes, or, poles, or roots result from specifying a model order higher than the actual physical number of modes in the analysis band. For those using the TDPoly approach, it seems that this over specification of the model order is required to ensure proper identification of those modes which correspond to the physical ones. These extra computational modes help to improve the estimates of the responses by accounting for data residual effects, nonlinearities, noise and inconsistency due to mass loading, nonstationarities and a few other issues. The effect that these computational modes have on the stability plot and the ability to interpret the results is to add complexity and confusion as to which are the proper modes to be extracted. In the worst case, the physical modes can be completely masked by the computation modes. While the use of various forms of the MIF and the implementation of the MCF can be used to help filter the stability diagram so that the physical modes are more readily observed, it can still be difficult to clearly observe all of the proper modes when modal density is high and wide frequency ranges are being used.

An example of a TDPoly stability plot which is complicated by the computational poles is shown in Figure 4, where noisy data and some modes with higher damping make the interpretation of the proper pole selection mode difficult. The test data was collected from a highly damped plate built by San Diego Composites. Two impact locations were used as references along with eight response locations. Damping values for all modes ranged from 4 to 8 percent of critical. Applying the MCF to visually filter some of the computational poles also precludes identification of some of the more heavily damped modes as seen in Figure 5. Due to the nature of the modal confidence factor, more heavily damped and higher frequency modes often have lower modal confidence values because the time history response of those signals dies off relative to other lower frequency and less damped

modes. The time domain least-squares solution also does a better job of fitting modes with longer response times in the time domain – those with lower damping and lower frequencies.

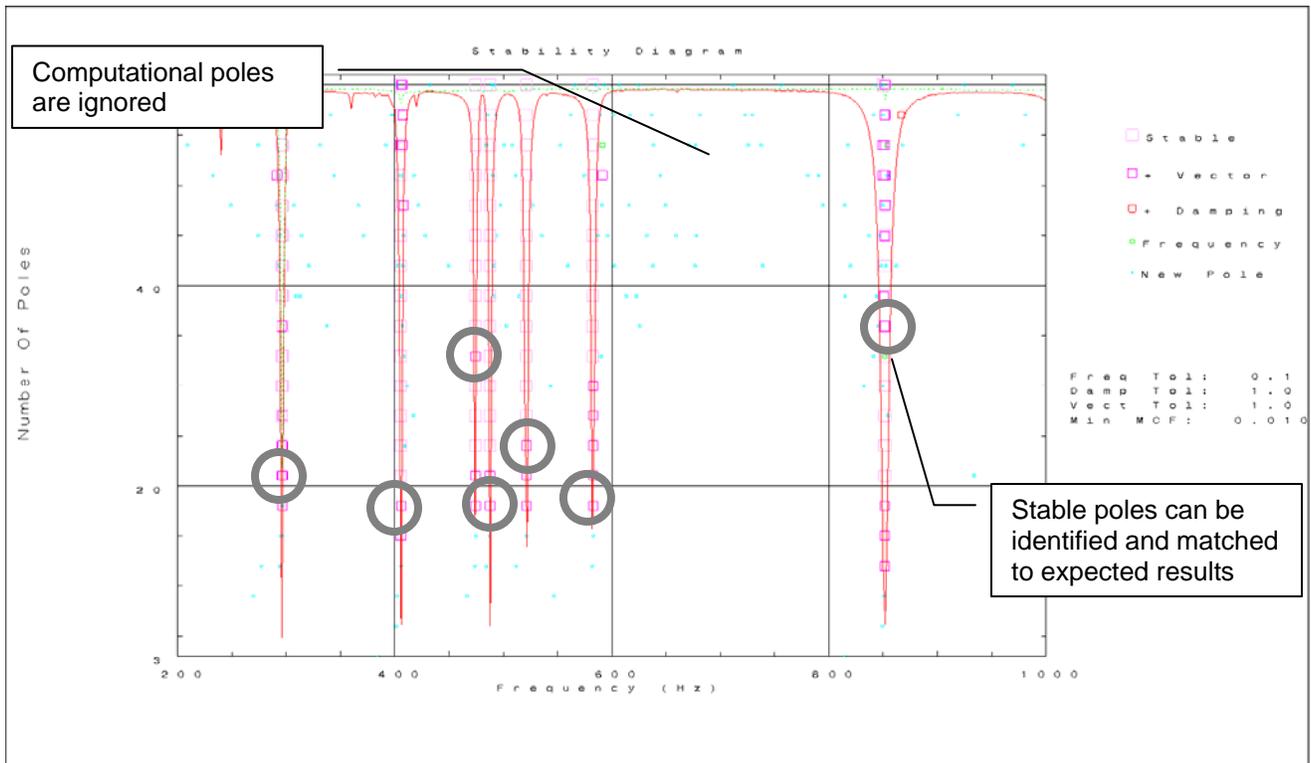


Figure 1. Stability plot combined with Multivariate Mode Indicator Function for parameter selection.

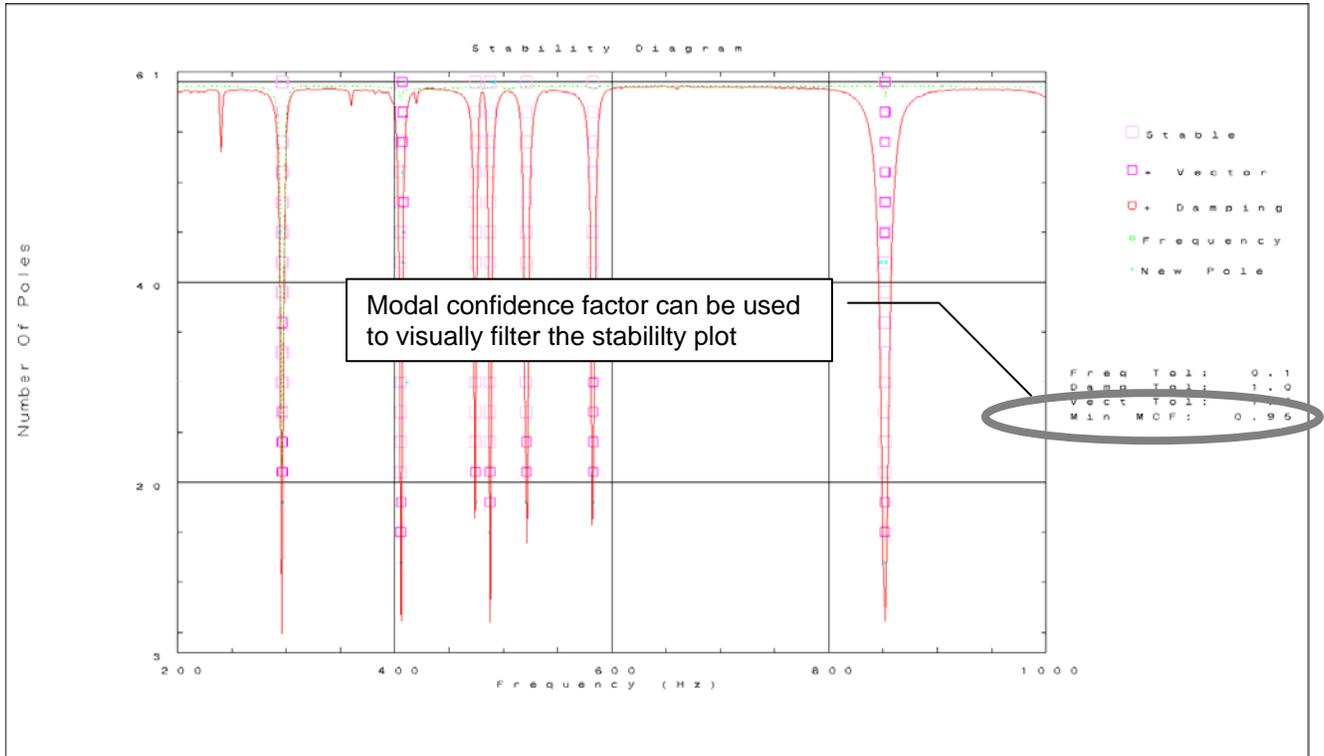


Figure 2. Stability plot visually filtered using MCF.

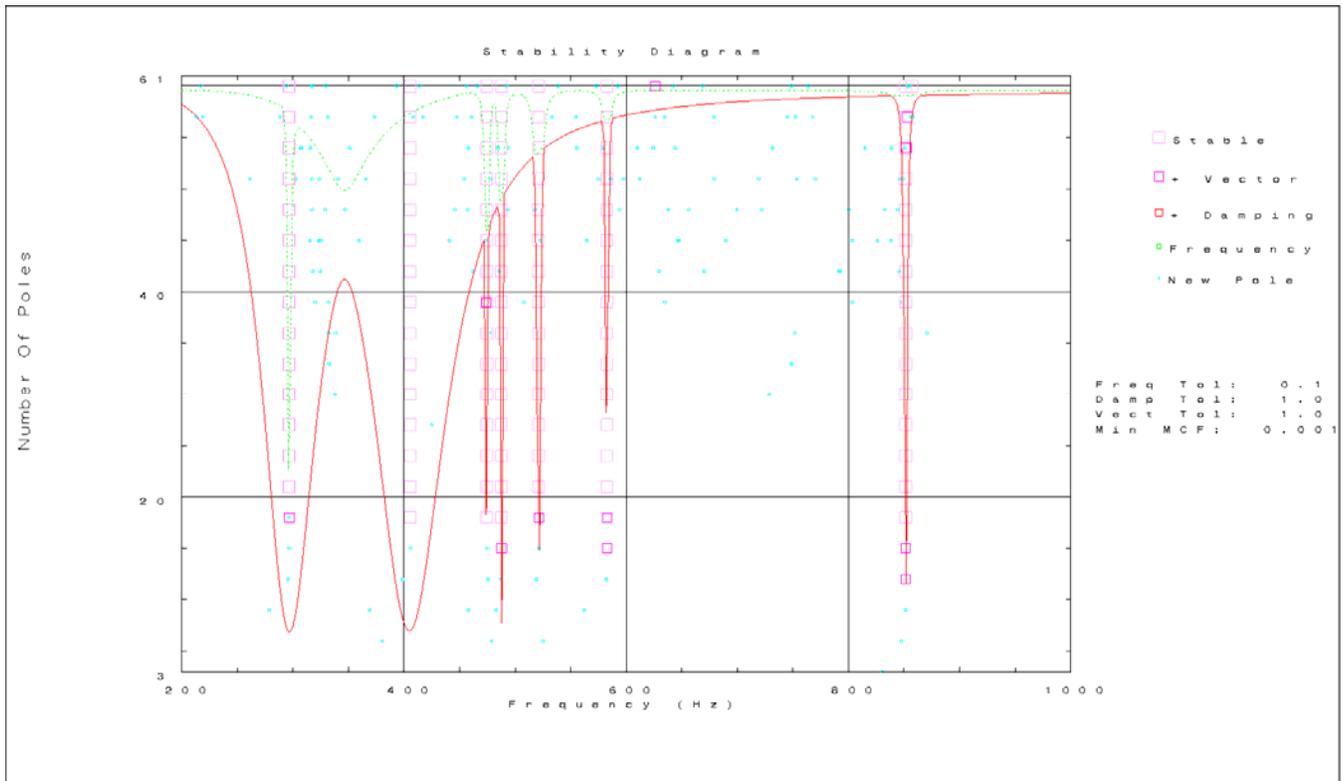


Figure 3. In the absence of sensor noise, heavily damped modes can be identified using polyreference.

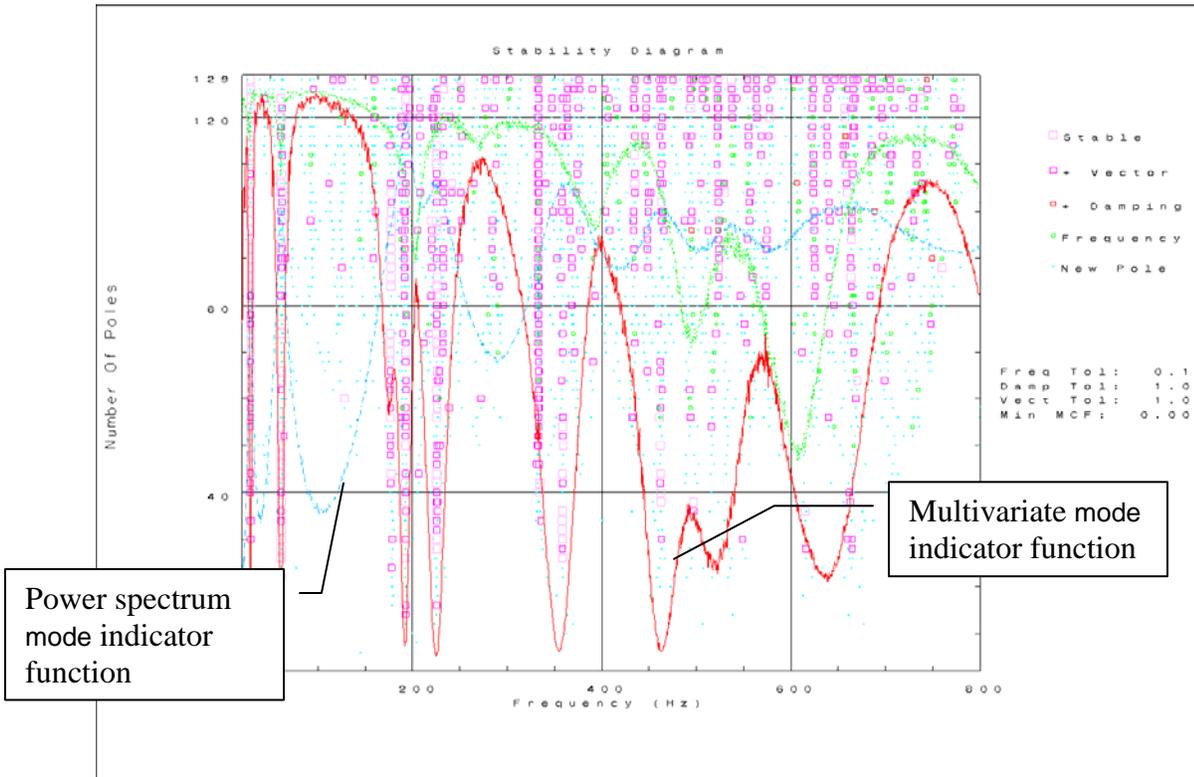


Figure 4. More computational poles result with noisy data and modes with varying or high damping.

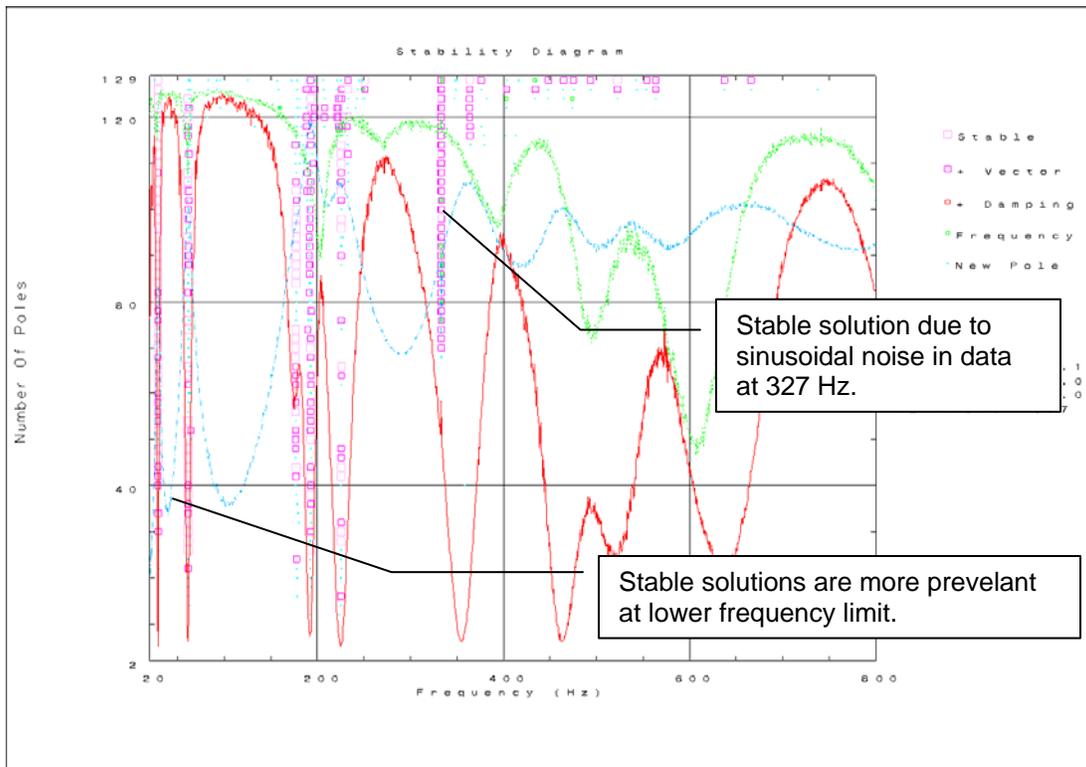


Figure 5. MCF helps visually filter some computational poles but also filters out heavily damped modes. Modal confidence is higher for lower frequency, more lightly damped modes.

## A LAPLACE DOMAIN ESTIMATOR

Nonetheless, even with some of the afore mentioned limitations, TDPoly still retained a dominant position, and was the tool of choice for most modal test engineers through the 1990's. There were few changes made during that decade and no real new modal parameter estimation tools were developed. A new method, introduced in 2006, called Alias Free Polyreference, or AFPoly [8], offers some improvements to TDPoly which can make modal parameter extraction easier and the visualization of results clearer, even in cases of mixed damping and high modal density.

AFPoly is a frequency domain parameter estimation tool that uses orthogonal polynomials in the continuous frequency domain that can be used to eliminate the effect of out of band mode effects on the estimates that can be generated when using discrete time domain estimators such as TDPoly. The out of band mode effects are eliminated by allowing for a solution outside the analysis frequency band. Figure 6 shows the stability diagram for the same set of test data as Figure 5 above using AFPoly. Stable poles are represented by the squares. The analysis band was from 20 to 800 Hz. This stability diagram shows four key features of the method. The first key is that the algorithm locates all of the poles in the test data without the use of a modal confidence factor to help interpret results. Second, evidence of the AFPoly method accounting for out-of band modes is clearly shown by the estimate of a significant mode near 880 Hz. Third, the method is less susceptible to noise than the TDPoly method. This stability diagram was generated including the 327 Hz noise in the solution, but it can also be removed from the set of analysis frequencies if it were to cause a significant error. Fourth, a significant number of computational poles are calculated near the edge of the frequency band.

A close-up of the lower analysis band shows the effect of the computational poles near the edge of the analysis band. There are vertical and slightly drifting computational poles. These poles shown in Figure 7, however, do not affect the user's ability to select a stable solution. Contrary to the discrete time domain approach, the continuous frequency domain approach offers the user a better chance of fitting the higher frequency modes because there are more frequency lines of data at higher frequencies than at lower frequencies. This skews the least squares solution towards estimating the higher frequency modes more accurately.

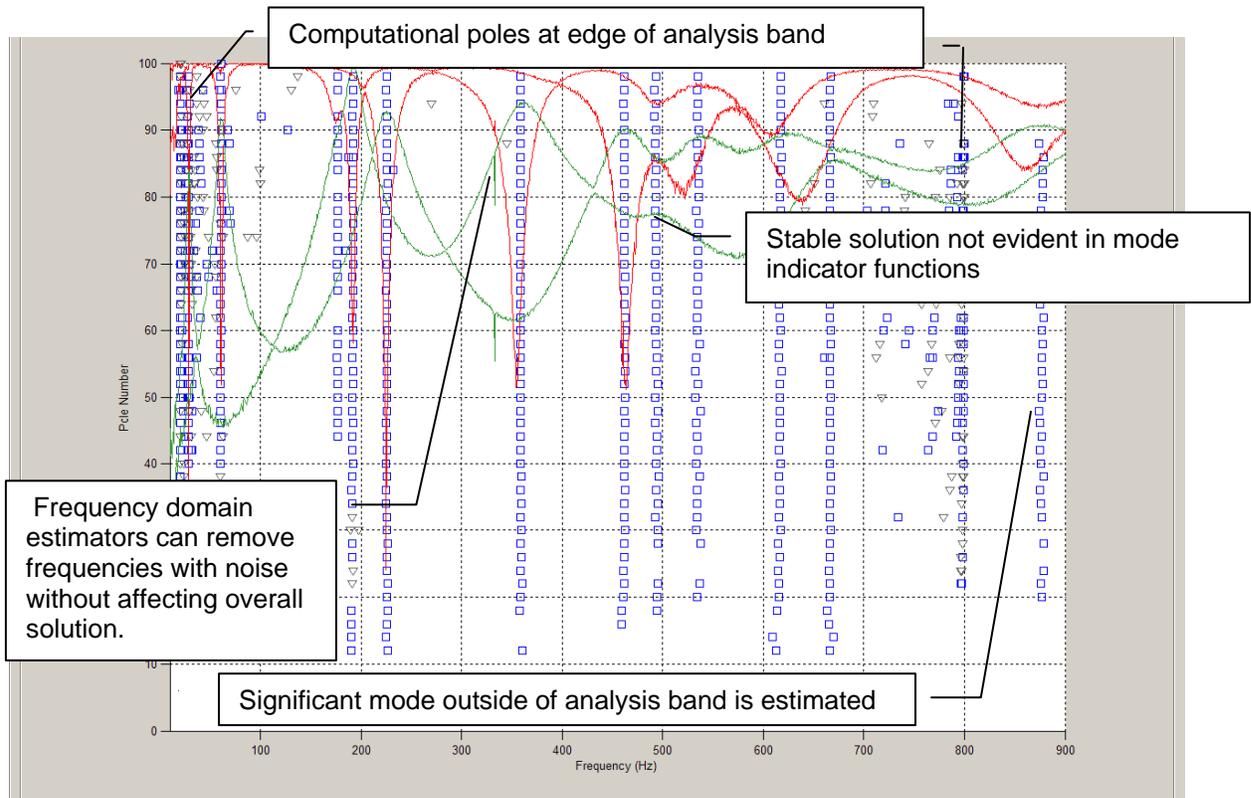


Figure 6. Example stability diagram with CMIF and MMIF overlays for analysis frequency band of 20 to 800 Hz.

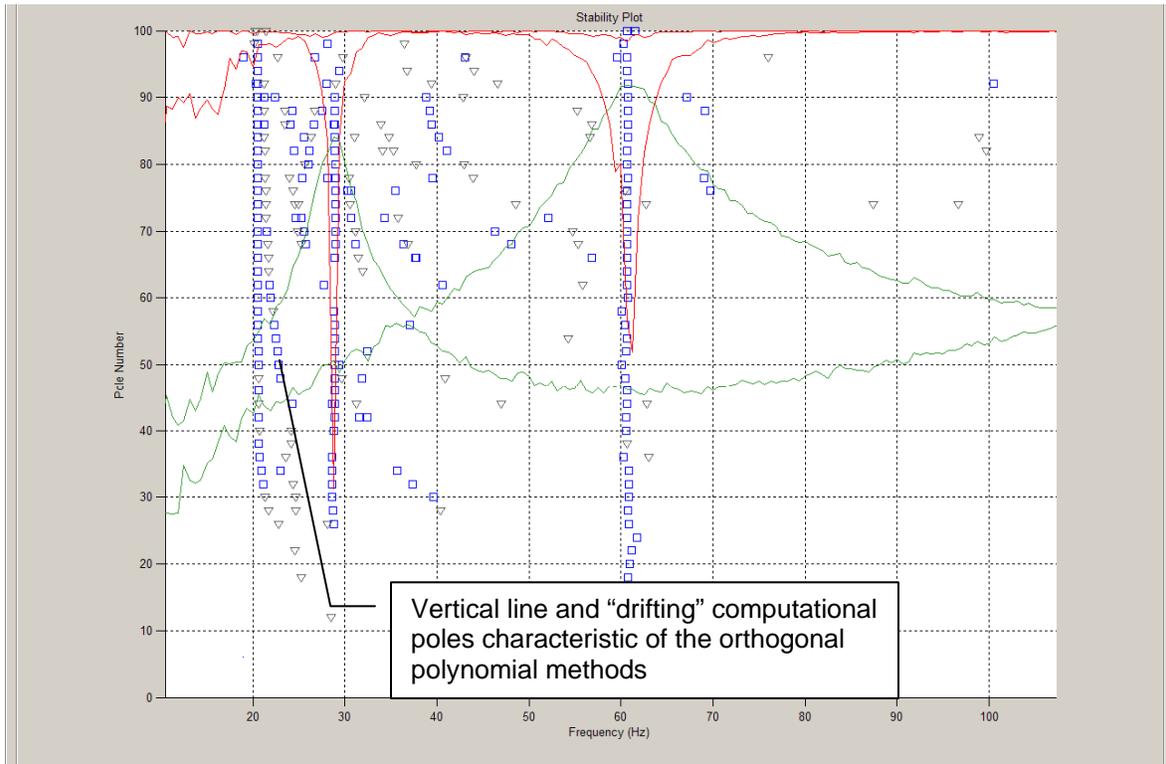


Figure 7. Close-up of lower bound shows computational poles.

## RECOMMENDED BEST PRACTICES

All parameter estimation methods work well with noise-free data. In the presence of noise, however, the AFPoly method excels at extracting modes with high damping or modes with uneven damping distribution over a wide frequency range. Modes at the upper end of the analysis band are often better-extracted because there are more frequency lines of information for a given mode. TDPoly excels at extracting lightly damped modes near the lower bounds of the analysis frequency because the least squares estimate of the time domain transform emphasizes lower frequency, lightly damped exponential decay.

## SUMMARY

Time Domain Polyreference has long held a position as the most commonly used and most robust parameter estimation tool in wide spread use. That may continue as TDPoly continues to be reliable and easy to use. Nonetheless, new methods that have been introduced in the past few years attempt to improve on ease of use and ease of results interpretation. AFPoly is one method which shows promise in its ability to cover a wide range of types of data while being easy to interpret the results. AFPoly appears to provide a reliable and complete modal parameter estimation tool which can improve on TDPoly in its ability to yield full frequency range results without limitations due to the degree of damping.

## ACKNOWLEDGEMENT

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