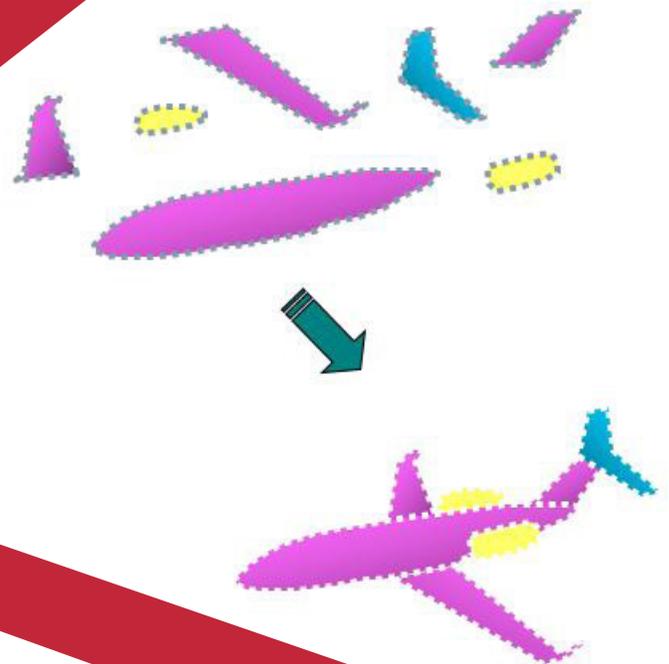


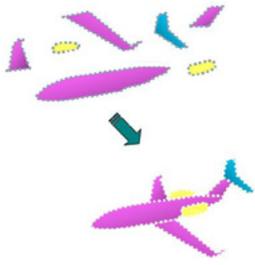
NX Nastran 10

**WHITEPAPER**

NX Nastran Model Reduction  
and Superelements:  
Theory and  
Implementation



# NX Nastran Model Reduction and Superelements: Theory and Implementation



Software:  
NX Nastran 10

## Overview

When modeling complex systems in NX Nastran, detailed models of components are often available. In many cases it is not practical to include the full model of every component in the system. NX Nastran superelements automate the process of creating reduced representations of components and efficiently assembling them into a system model. The purpose of this whitepaper is to provide background on superelements and describe the three primary ways of creating and using superelements in NX Nastran: main bulk data, part, and external superelements.

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# NX Nastran Model Reduction and Superelements: Theory and Implementation

## Reasons to Consider Using Superelements

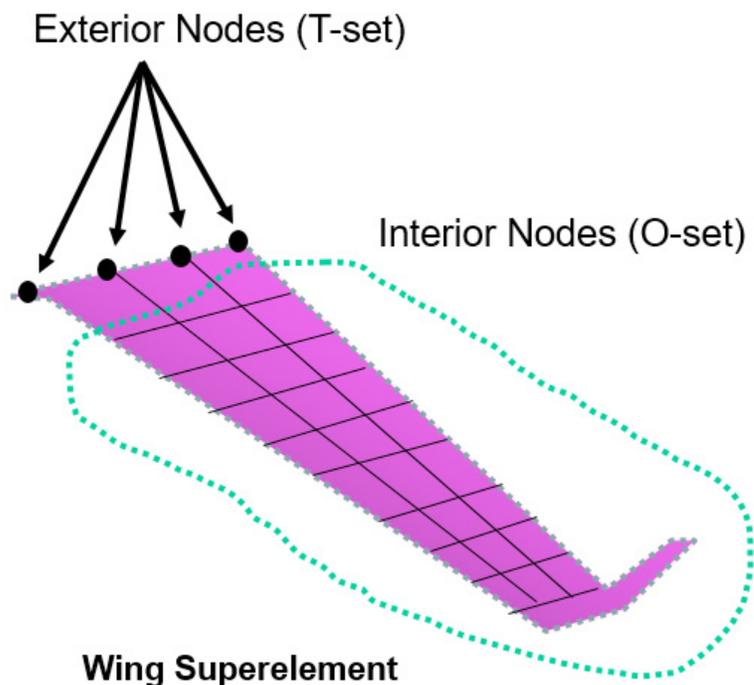
Below are a number of situations in which to consider using a superelement approach to develop a system model.

1. An assembly of all component models representing a system is too large
2. Some component models are proprietary
3. Design studies will be performed where most of the system remains unchanged
4. Nonlinearities in a system can be isolated to a small region
5. Models for a system do not follow unique numbering schemes

## Superelement Reduction

The concept behind superelement reduction is to take a finite element model (FEM) of a component and partition it into a set of interior and exterior nodes as illustrated in Figure 1. The FEM of the component is then mathematically reduced to just the exterior nodes, before assembling it into a model of the system. We will refer here to interior and exterior degrees of freedom (DOF), which are the three translations and three rotations at each node.

**Figure 1:** ►  
A superelement is partitioned into exterior and interior nodes, with the reduced model including only the exterior nodes.



## NX Nastran Model Reduction and Superelements: Theory and Implementation

The mathematical process used by NX Nastran to reduce a FEM to a small number of exterior nodes (or DOF) is referred to as static or Guyan reduction. The reason that it is called static reduction is that it depends only on the static solution (i.e., the stiffness matrix). The partitioning of the static solution is as follows:

$$\begin{bmatrix} K_{TT} & K_{TO} \\ K_{OT} & K_{OO} \end{bmatrix} \begin{Bmatrix} x_T \\ x_O \end{Bmatrix} = \begin{Bmatrix} f_T \\ f_O \end{Bmatrix} \quad (1)$$

The interior deflections  $\{x_O\}$  can be calculated from the exterior deflections  $\{x_T\}$  and the interior forces  $\{f_O\}$  as follows:

$$\{x_O\} = -[K_{OO}]^{-1}[K_{OT}]\{x_T\} + [K_{OO}]^{-1}\{f_O\} \quad (2)$$

The first term, multiplying the exterior deflections, is often referred to as the constraint shapes since they are the shapes that would be calculated by deflecting each exterior DOF while holding all others fixed. These shapes can then be used to reduce the equations down to just the exterior DOF as follows:

$$[K_{TT} - [K_{TO}][K_{OO}]^{-1}[K_{OT}]]\{x_T\} = \{f_T\} + [K_{TO}][K_{OO}]^{-1}\{f_O\} \quad (3)$$

The full equations for the component, therefore, have been replaced with equations on only the exterior DOF. This means that for a component that might potentially be represented by millions of DOF, but with only a few dozen exterior DOF, the equations can be reduced to just the few dozen DOF.

While the static reduction is developed entirely using the stiffness matrix, it can be applied to the mass matrix as well. This captures the rigid body mass but does not capture internal dynamics, meaning modes of vibration that would be experienced if all exterior DOF were held fixed. To capture the interior dynamics, NX Nastran superelements introduce a concept called component mode synthesis (CMS). CMS appends additional generalized DOF describing the internal modal dynamics of the component. Using NX Nastran terminology, these are referred to as the Q-set or  $\{x_Q\}$ . This adds an additional term to Eq. 2:

$$\{x_O\} = -[K_{OO}]^{-1}[K_{OT}]\{x_T\} + [\Phi_O]\{x_Q\} + [K_{OO}]^{-1}\{f_O\} \quad (4)$$

Note that  $\{x_Q\}$  is not a subset of the DOF of the originally unreduced component FEM but rather additional generalized DOF that are associated with component mode shapes  $[\Phi_O]$ . If the component modes are calculated with the exterior DOF held fixed, the resulting reduced model is referred to as either a Craig-Bampton (CB) or Hurty/Craig-Bampton (HCB) model. NX Nastran provides a minor generalization of HCB models since selected DOF can be left free, but for

## NX Nastran Model Reduction and Superelements: Theory and Implementation

the purposes of this whitepaper we will consider only the HCB case. The reduced equations for an HCB model are

$$\begin{bmatrix} M_{TT} & M_{TQ} \\ M_{QT} & M_{QQ} \end{bmatrix} \begin{Bmatrix} \ddot{x}_T \\ \ddot{x}_Q \end{Bmatrix} + \begin{bmatrix} K_{TT} & 0 \\ 0 & K_{QQ} \end{bmatrix} \begin{Bmatrix} x_T \\ x_Q \end{Bmatrix} = \begin{Bmatrix} \{f_T\} + [K_{TQ}][K_{QQ}]^{-1}\{f_Q\} \\ [\Phi_Q]^T \{f_Q\} \end{Bmatrix} \quad (5)$$

While CMS is an approximate solution to the dynamics of a component, it is typically very accurate as long as the component modes are calculated to 1.5–2 times the frequency range of the system modes.

Both static and dynamic (CMS) reductions are supported by NX Nastran superelements. Static reductions are appropriate for static solutions and also for dynamic solutions where the internal dynamics of a component are unimportant or well above the frequency range of the system analysis, or where the internal dynamics are deliberately neglected. Dynamic reduction is applicable to a very wide range of dynamic problems, only requiring that the component remain linear.

### General Rules for Superelement Reduction

The reduction of a component model to a superelement can dramatically reduce the number of DOF (or equations) required to represent that component. However, the reduction step itself can be computationally expensive, and the resulting matrices are typically much denser (many fewer zero terms) than the original equations. Because of this, superelements only provide an advantage in practice when the number of interface DOF,  $\{x_i\}$ , is *much* less than the number interior DOF,  $\{x_o\}$ . This requires that the interfaces for superelements be chosen carefully to constitute as small a number of DOF as possible. Typically, having more than about 100 nodes on the exterior of a superelement results in poor numerical behavior, though there are exceptions.

### Superelements and Residual Vectors

Equations (2) and (4) not only provide means for reducing the equations of a component but also for recovery of internal deflections,  $\{x_o\}$ . For a dynamically reduced component, these equations are typically referred to as mode displacement data recovery, since the internal displacements are recovered using the modal displacements rather than the modal accelerations. Mode displacement data recovery suffers from a well-known modal truncation issue. Even though the modes may accurately represent the dynamic response of the component in a particular frequency range, they do not necessarily capture the internal quasi-static response. That is the response when the component

# NX Nastran Model Reduction and Superelements: Theory and Implementation

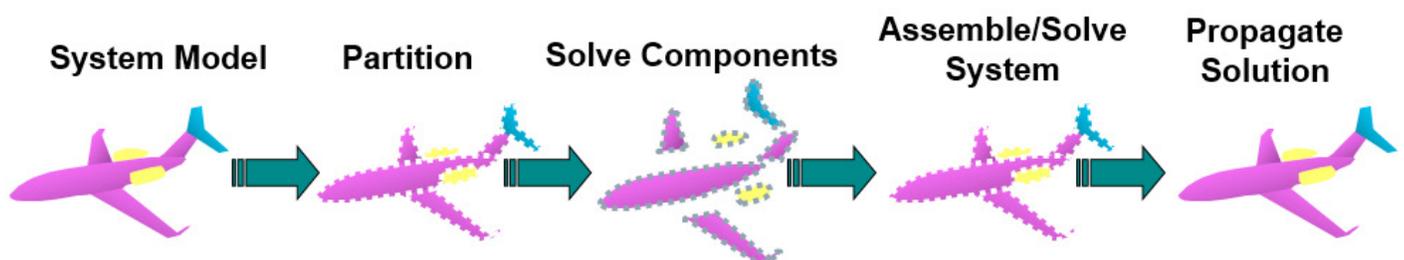
experiences a steady-state component in an applied load or acceleration. To correct for this, NX Nastran adds residual vectors that capture the quasi-static contribution of all the truncated modes to the applied loads. By default, NX Nastran calculates residual vectors for internally applied loads and for six rigid body acceleration components. Because of this, the number of modes used to represent a component with residual vectors is equal to the number of fixed interface modes in the frequency range of interest, plus the number of internally applied loads, plus six for rigid body acceleration. Residual vectors are crucial for the accurate recovery of all internal responses including displacements, element forces, stresses and strains, grid point forces, and multipoint constraint forces, and while it is possible to deactivate them, this is not recommended.

## Main Bulk Data Superelements

NX Nastran offers three different methods for generating superelement models; main bulk data superelements, external superelements, and part superelements. While the mathematics of superelement reduction are identical across all three methods, the user interface is very different, and each one may be the best choice depending on the problem at hand. The most mature of these methods is referred to as “main bulk data superelements,” because all the superelements reside in a single NX Nastran model (the main bulk data). In this case, the user enters a small number of additional inputs that specify how the model is partitioned. NX Nastran automatically partitions the model, reduces each of the components, assembles and solves the system, and then propagates the solution to the interior of all the superelements, as illustrated in Figure 2.

**Figure 2:** ▼

Main bulk data superelements take a top-down approach. The full model is partitioned, each component solved, and the system is then reassembled and solved and the results propagated into the individual superelements, potentially in a single run.



# NX Nastran Model Reduction and Superelements: Theory and Implementation

A main bulk data superelement model of a system is identical to a non-superelement model, with the further restriction that all element numbers must be unique across the entire model. The partitioning into superelements is specified by identifying the interior nodes on SESET cards. By using compact numbering ranges a single SESET card can be used to identify each superelement. That is all that is required for statically reduced superelements. Dynamically reduced superelements must also have DOF defined for component modes (typically SPOINTs). The structure of a typical main bulk data superelement input file is illustrated in Figure 3.

**Figure 3:** ▶  
Structure of a main bulk data superelement input deck.

```

SOL      100      $ Normal modes
CEND
DISP (PLOT) = ALL
SUBCASE 100
  SUPER = 100
  METHOD = 70
SUBCASE 1000
  METHOD = 70
  SPC = 10
BEGIN BULK
$
EIGRL  70      1.0      70.0
EIGRL  100     1.0      100.0
$
$ Spacecraft bulk data
INCLUDE 'gpsc.blk'
$
$ Superelement Definition
SESET  100     1      THRU   18
SESET  100     40     THRU   54
SESET  100     51     THRU  1010
SPOINT 1001   THRU   1010
SEQSET1 100    0      1001   THRU   1010
$
ENDDATA

```

This model contains a single superelement (SE 100) whose internal nodes are 1–18, 40, and 51–54. All other nodes are in the residual structure, which consists of all parts of the model that are not in a superelement.<sup>11</sup> The first SUBCASE refers to SE 100 and specifies a component mode cutoff of 150 Hz, while the second SUBCASE refers to the residual and specifies a system mode cutoff of 70 Hz. The SPOINT and SEQSET1 cards specify ten generalized DOF that are used to store the modal DOF (q-set). Removal of the SESET, SPOINT, and SEQSET1 cards would revert this model back to a FEM of the full system without any superelement reductions.

One of the advantages of a main bulk data superelement model is that the entire process can be executed in a single run with minimal alterations to the non-superelement input file. However, it is also possible to solve each superelement separately and save results in individual databases by using the SExx Case Control inputs to control the steps taken in each run. This is described in a later section.

<sup>11</sup> The term “residual structure” in the context of superelements refers to the partition of the FEM that remains after all superelements are reduced. It may or may not contain any elements, but it has nothing to do with “residual vectors,” which are used to address modal truncation issues.

# NX Nastran Model Reduction and Superelements: Theory and Implementation

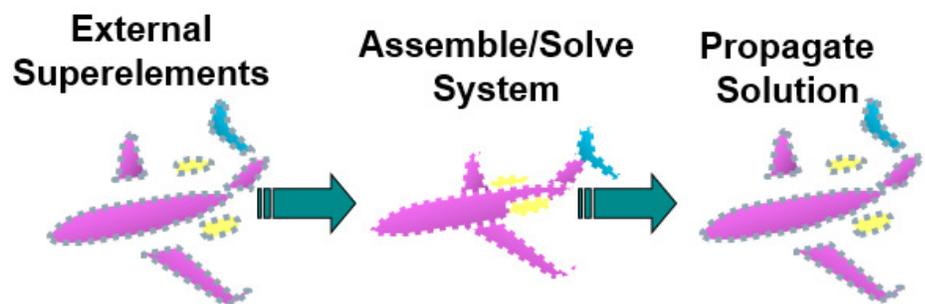
The primary disadvantage of the main bulk superelement method is that it requires that the entire model be made available to NX Nastran with unique grid and element numbering. Often the reason for using superelements is that the component FEMs are developed by independent organizations that do not necessarily adhere to unique numbering schemes and who may want to hide the component internal details. For this case, an external superelement approach may be preferred.

## Generating External Superelements

External superelements address the case where component FEMs are developed by different organizations who may only want to share the reduced models without making the internal details visible. In this case a bottom-up approach is taken, as illustrated in Figure 4.

**Figure 4:** ►

External superelements take a bottom-up approach to assembling a system. Each external superelement is solved independently.



Rather than presenting Nastran with an input file describing the entire system and then providing SESET cards to partition that system, each superelement is instead treated as a separate FEM and reduced to a set of matrices in a separate run. Only the reduced representation, consisting of the interface nodes and the reduced matrices, is passed on to the system model. The organization doing the reduction, therefore, only needs to know the location of the exterior nodes where their structure will interface with the system, and the system analysis organization only needs to have the exterior nodes and a matrix representation of each external superelement. This is ideal for situations such as coupled loads analysis (CLA), where entirely different organizations are responsible for different components of the system, but it can also provide advantages where a library of pre-reduced models of various components are desired for system-level analysis.

## NX Nastran Model Reduction and Superelements: Theory and Implementation

To define an external superelement, the user simply identifies the interface DOF on ASET cards<sup>2</sup> and optionally provides generalized DOF for dynamic reduction using SPOINT and QSET cards. Every DOF that is not identified on an ASET is treated as an interior DOF. A sample external superelement generation input file is illustrated in Figure 5.

**Figure 5:** ►

Example of an external superelement generation.

```

ASSIGN OUTPUT4='gpsec.op4' UNIT=11 FORMATTED
SOL      103      $ Normal modes
CEND
METHOD = 70
SET 10 = 18, 40, 42
DISP(PLOT) = 10
SET 20 = 87, THRU, 90
FORCE(PLOT) = 20
EXTSEOUT(MATOP4=11, EXTTD=100)
BEGIN BULK
EIGRL   70              70.0
$ Spacecraft bulk data
INCLUDE 'gpsec.blk'
$ Boundary Dof at base of four feet
BSET1  123456 44      45      48      49
$ Component mode DOF (150)
SPOINT 101      THRU  150
QSET1  0        101      THRU  150
ENDDATA

```

This generates an external superelement with 24 exterior DOF (six at each of four nodes) and up to 150 modal DOF (including residual vectors). The EXTSEOUT card in Case Control specifies that the matrices be written in OUTPUT4 format to Unit 11 and that the superelement be labeled as SE 100. Data recovery matrices will be generated for the displacement of nodes 18, 40, and 42 and forces in elements 87–90.

External superelements do support internal data recovery as illustrated in Figure 5, though all requests must be made at the time that the superelement is reduced in order to support system results. The output requests can include displacement, velocities and accelerations (include a DISPLACEMENT request for those nodes in the reduction), element forces, stresses and strains, grid point forces, and MPC and SPC forces. These are converted to data recovery matrices, which are used after the system solution to recover the internal results. As long as residual vectors are included, the accuracy of the internal responses is typically excellent.

Internal loads can also be applied to external superelements, though they need to be specified at the time of reduction and an SELOAD card needs to be used to associate external superelement loads to system-level loads. Because of this, the use of internal loads is significantly more complicated when using external superelements, and it is usually preferable to avoid this as much as possible.

<sup>2</sup> Exterior DOF can also be specified on BSET, CSET, BNDFIX, and BNDFREE cards for dynamic reduction. This affects the boundary conditions used for the component mode calculation.

# NX Nastran Model Reduction and Superelements: Theory and Implementation

NX Nastran offers a large number of formats for storing the reduced matrices. These include two database formats (DMIGDB and MATDB), an OUTPUT2 format (DMIGOP2), an OUTPUT4 format (MATOP4), and a DMIG format (DMIGPCH). The most popular formats are DMIGOP2, MATOP4, and DMIGPCH. DMIGOP2 is a binary format that stores both the matrices and the exterior geometry. MATOP4 (Figure 5) stores the matrices in OUTPUT4 format (either binary or formatted) and the exterior geometry in standard bulk data. DMIGPCH stores the matrices in a DMIG format and the exterior geometry in standard bulk data. The formatted versions of MATOP4 and DMIG are probably the most commonly used because the formatted matrices are easy to interpret and read into other programs such as MATLAB if necessary. The MATOP4 format, in particular, forms the basis of most CLA, while DMIG<sup>3</sup> can be very easily passed to another model without requiring a superelement license and is therefore the most portable format.

## Part Superelements

The previous section describes the generation of an external superelement reduced model, but to use an external superelement, a part superelement model is required. An NX Nastran part superelement model is one where the input file is divided into sections (or parts) using BEGIN SUPER nnn cards, where nnn is the superelement number. All bulk data between BEGIN BULK and the first BEGIN SUPER card represents the residual structure, all bulk data between BEGIN SUPER mmm and BEGIN SUPER nnn represents superelement ID mmm, etc. This is a bottom-up approach as illustrated in Figure 4. Each part superelement can be a previously reduced external superelement, or it can be a FEM that will be reduced by Nastran during the execution of the part superelement run. A typical part superelement input file is illustrated in Figure 6.

**Figure 6:** ▶  
Structure of a part superelement input deck.

```

ASSIGN INPUT4-'se200.op4' UNIT=11
SOL 103 $ Normal modes
CEND
DISP (PLOT) = ALL
METHOD = 70
SPC = 10
BEGIN BULK
PARAM WTMASS .00259
SPC1 10 123456 44 45 48 49
SENQSET 100 20
EIGRL 70 70.
INCLUDE 'gpbus.blk'
SENHUK 200 EXTUP4 11
BEGIN SUPER 100
EIGRL 70 150.
INCLUDE 'gpsecant.blk'
INCLUDE 'se200.pch'
ENDDATA

```

<sup>3</sup> The primary disadvantage of the DMIG option is that the matrices are entered with only 10 digits of precision, while the other options all use 16 digits of precision. The missing digits often result in artificial grounding that is picked up by the GROUNDCHECK command in NX Nastran.

# NX Nastran Model Reduction and Superelements: Theory and Implementation

This model consists of a residual structure (gpbus.blk) and two part superelements (gpsecant.blk and se200.pch). The residual and SE 100 are each valid models in their own right. Each contains all required grids, elements, properties, coordinate systems, parameters, etc. In fact, in this case, the residual uses a PARAM WTMASS of 0.00259, while SUPER 100 uses the default value of 1.0. Each superelement also has its own solution cards such as an EIGRL card to specify a frequency cutoff. In this case METHOD=70 is called out in Case Control, but this refers to a 70 Hz cutoff for the system results and a 150 Hz cutoff for the superelement. The SENQSET card automatically assigns twenty generalized (q-set) DOF to SE 100 for component modes. In this case, SE 200 is an external superelement in MATOP4 format. The OUTPUT4 file is assigned to UNIT 11 at the top of the deck, the superelement is identified in the bulk data using an SEBULK card, and the BEGIN SUPER 200 and all other required input are in the se200.pch file, which was generated by NX Nastran when reducing SE 200 as an external superelement.

Because part superelements are placed in different parts of the input file, there is no requirement for unique numbering from superelement to superelement. In fact, the connectivity between superelements depends on geometrically coincident nodes rather than numbering and is fully automated.<sup>4</sup> Unlike external superelements, however, the entire model is available to NX Nastran so all reductions, the system solution, and data recovery can be performed in a single run, though that is not necessary. Another advantage of making all the bulk data available for each superelement is that application and updates of loads or additional data recovery requests are all handled automatically and do not require manual reprocessing of external superelements.

## Using Restart with Superelements

When using external superelements, each superelement is reduced independently and then assembled into a system. Because of this, external superelements are typically only reduced once. For both main bulk data and part superelements, on the other hand, the entire process can be executed in a single run. While this may seem like an advantage, the computational advantage of superelements can typically only be realized by avoiding reducing the superelements every time. Because of this, it is typically critical to use existing databases to avoid repeating superelement reductions. There are two ways to do this. The first is to use a DBLOCATE to attach an existing database with a superelement solution. This is illustrated in Figure 7.

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<sup>4</sup> NX Nastran offers a number of options for controlling how part superelements are connected. These vary from a fully automated approach based on finding nodes within some distance of each other to completely manual connection using SECONCT cards.

# NX Nastran Model Reduction and Superelements: Theory and Implementation

**Figure 7:** ►

Superelement restart using DBLOCATE.

```

ASSIGN MASTER = `resid.MASTER` DELETE
ASSIGN DBALL = `resid.DBALL` DELETE
ASSIGN SE100 = `se100.MASTER`
DBLOCATE DATABLK=* LOGICAL=SE100
SOL 103
CEND
SET 1000 = 0, 200
SEALL = 1000
...
BEGIN BULK
...
ENDDATA

```

In this case, the database se100.MASTER was created in a previous run and contains a reduced model of SEID 100. The SEALL = 1000 card indicates that superelement operations will only be performed on SEID 0 (the residual) and SEID 200, assuming that SEID 100 has already been reduced. This will cause NX Nastran to locate all required datablocks for SEID 100 in the existing database rather than repeating the reduction.

An alternative method is to use NX Nastran's automated RESTART capability. RESTART works independently of superelements and works out what parts of a previous solution need to be repeated based on changes in the input file. It is particularly powerful in the context of superelements because if nothing has changed in a superelement, the reduction will not be repeated, but if there are changes to a superelement such as new loads or data recovery requests, the minimum steps required to handle these will be automatically applied. There are two ways of performing an automated RESTART in NX Nastran. The first uses just an existing database and a RESTART card. This will create a new "version" in the database with the updated results. The downside of this approach is that if there are any errors in the restart run, the database can very easily become corrupted and unusable for further restarts. As such, a better approach is to treat the original database as "read-only" and save new results in a new database file. An example of this is illustrated in Figure 8.

**Figure 8:** ►

Superelement restart using RESTART LOGICAL.

```

ASSIGN MASTER = `resid.MASTER` DELETE
ASSIGN DBALL = `resid.DBALL` DELETE
ASSIGN SE100 = `se100.MASTER`
RESTART LOGICAL=SE100
SOL 103
CEND
...
BEGIN BULK
...
/ ,1,99999999
...
ENDDATA

```

## NX Nastran Model Reduction and Superelements: Theory and Implementation

In this case, NX Nastran will compare the updated and original Case Control and Bulk Data on a superelement-by-superelement basis and determine what, if anything, has changed in each superelement. It will automatically decide which steps on which superelements need to be repeated. The /,1,9999999 card is required to delete the old bulk data and replace it with the new. Without this card, NX Nastran will append the new bulk data to the old and generate multiple errors due to duplicate cards. Alternatively, if no changes are made to Bulk Data, or cards are only added and not replaced, only the new cards need to be included and /,1,9999999 is left out.

### Conclusion

If used correctly, superelements can provide a very powerful tool for analyzing complex systems or sharing component models among organizations. NX Nastran provides three different ways of developing a superelement model, each with its own advantages and disadvantages: main bulk data superelements, external superelements, and part superelements. Much more information can be found in the Superelement User's Guide found with the NX Nastran documentation or by taking a two-day superelement course taught by ATA Engineering.

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